Module 4b: Water Distribution System Design

Hardy Cross Method

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Hardy Cross Method

- Used in design and analysis of water distribution systems for many years..
- Based on the hydraulic formulas we reviewed earlier in the term.

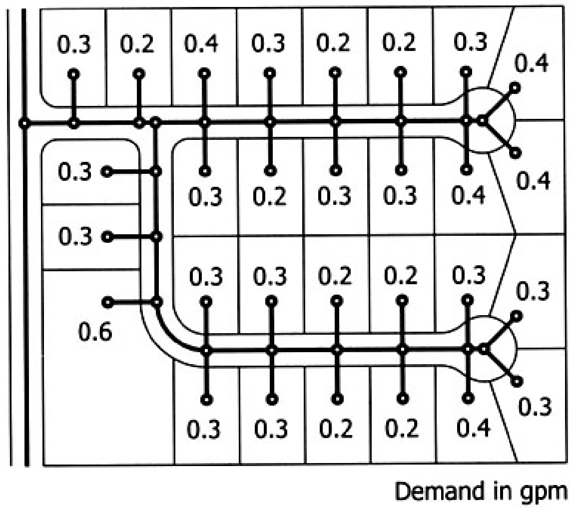
For Hardy-Cross Analysis:

- Water is actually removed from the distribution system of a city at a very large number of points.
- Its is not reasonable to attempt to analyze a system with this degree of detail
- Rather, individual flows are concentrated at a smaller number of points, commonly at the intersection of streets.
- The distribution system can then be considered to consist of a network of nodes (corresponding to points of concentrated flow withdrawal) and links (pipes connecting the nodes).
- The estimated water consumption of the areas contained within the links is distributed to the appropriate nodes

Network model overlaid on aerial photograph

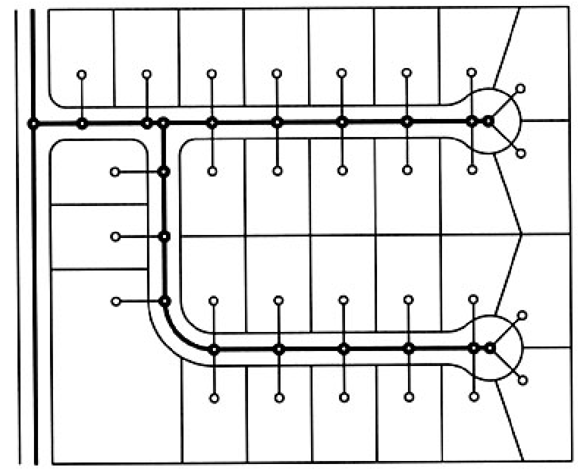
(Walski, *et al.* 2004 figure 3.4)

Skeletonization - An all-link network



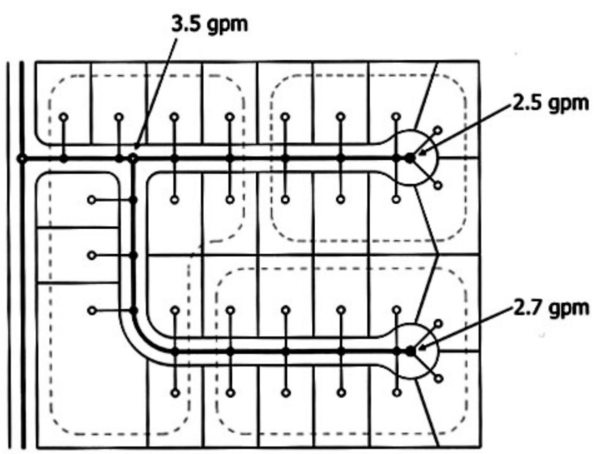
(Walski, *et al.* 2004 figure 3.32)

Minimal skeletonization



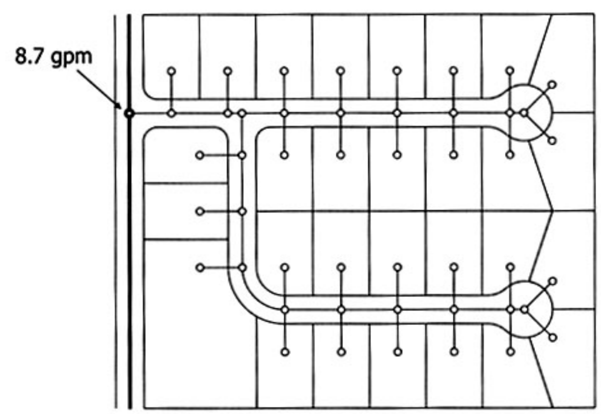
(Walski, *et al.* 2004 figure 3.33)

Moderate skeletonization



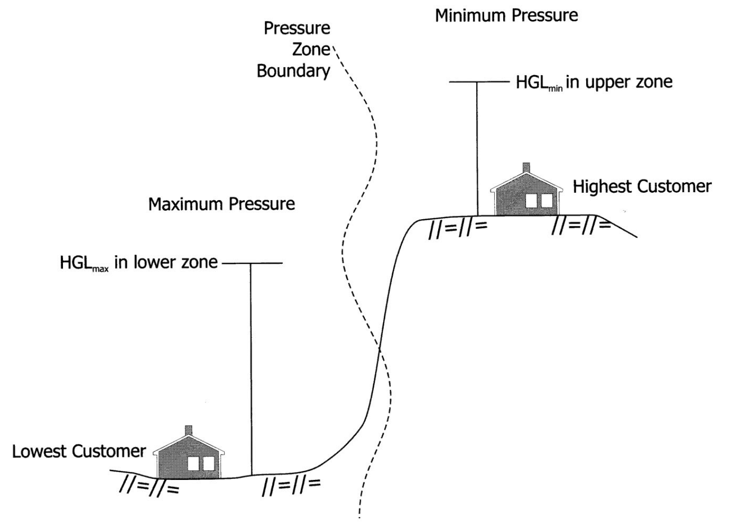
(Walski, *et al.* 2004 figure 3.34)

Maximum skeletonization



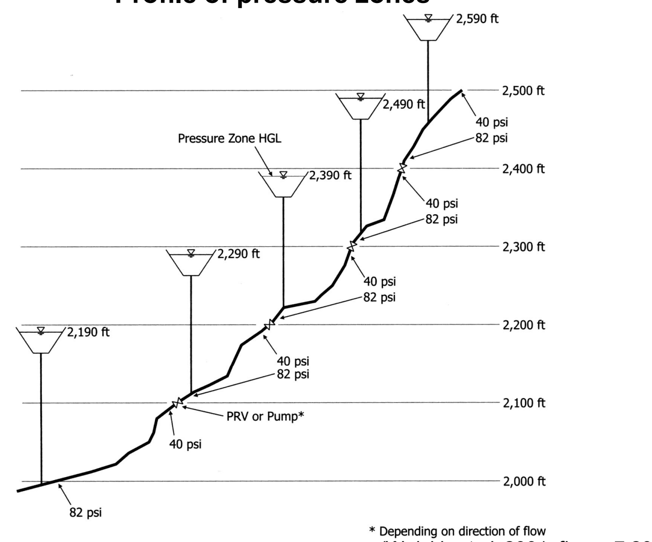
(Walski, *et al.* 2004 figure 3.35)

Customers must be served from separate pressure zones



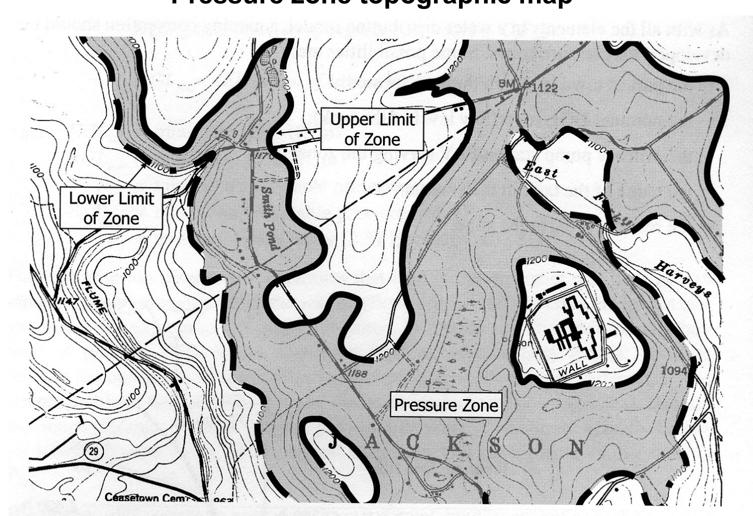
(Walski, *et al.* 2001 figure 7.17)

Profile of pressure zones



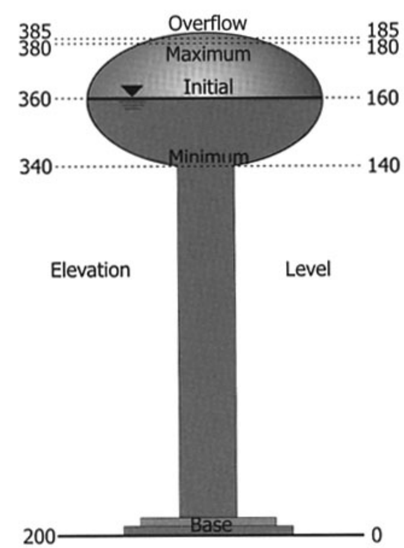
(Walski, *et al.* 2001 figure 7.20)

Pressure zone topographic map



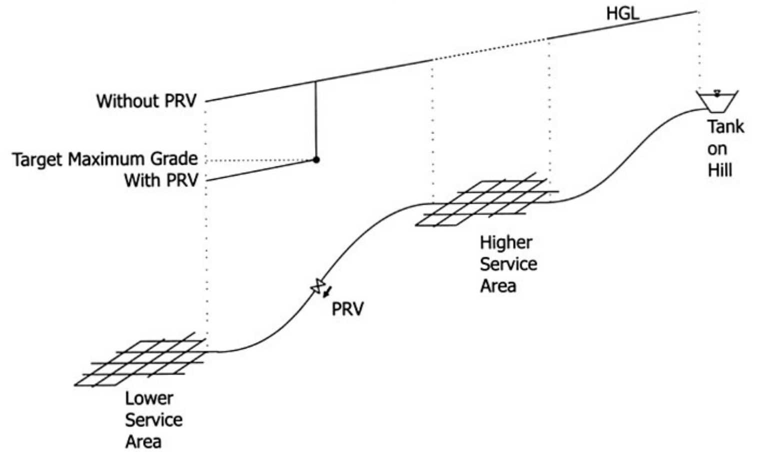
(Walski, *et al.* 2001 figure 7.21)

Important tank elevations



(Walski, *et al.* 2001 figure 3.10)

Schematic network illustrating the use of a pressure reducing valve



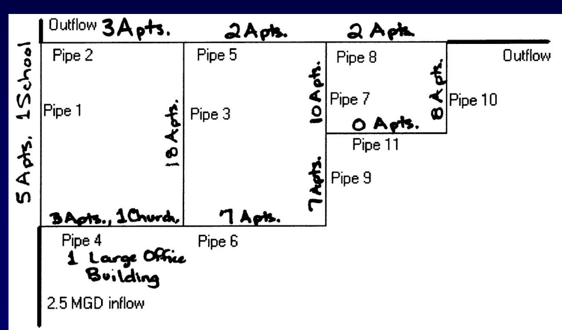
(Walski, *et al.* 2001 figure 3.29)

Hardy-Cross Method of Water Distribution Design

- Definitions
  - *Pipe sections or links* are the most abundant elements in the network.
    - These sections are constant in diameter and may contain fittings and other appurtenances.
    - Pipes are the largest capital investment in the distribution system.
  - *Node* refers to either end of a pipe.
    - Two categories of nodes are *junction nodes* and *fixed-grade nodes*.
    - Nodes where the inflow or outflow is known are referred to as junction nodes. These nodes have lumped demand, which may vary with time.
    - Nodes to which a reservoir is attached are referred to as fixed-grade nodes. These nodes can take the form of tanks or large constant-pressure mains.

Steps for Setting Up and Solving a Water Distribution System using the Hardy-Cross Method

1. Set up grid network to resemble planned flow distribution pattern.



Steps for the Hardy-Cross Method

2. Calculate water use on each street (including fire demand on the street where it should be located).

Street Number	Building Description	Without Fire Demand		With Fire Demand (worst building)	
		MGD	ft^3/sec	MGD	ft^3/sec
1**	5 A, 1 S	0.059	0.092	2.00	3.10
2	3 A	0.019	0.030	0.019	0.030
3	18 A	0.12	0.18	0.12	0.18
4	3 A, 1 O, 1 C	0.056	0.086	0.056	0.086
5	2 A	0.013	0.020	0.013	0.020
6	7 A	0.045	0.070	0.045	0.070
7	10 A	0.065	0.100	0.065	0.100
8	2 A	0.013	0.020	0.013	0.020
9	7 A	0.045	0.070	0.045	0.070
10	8 A	0.052	0.080	0.052	0.080
11	No buildings	0.0	0.0	0.0	0.0

Steps for the Hardy-Cross Method

3. Add up the flow used in the neighborhood without fire demand and distribute it out the nodes where known outflow is required. Repeat for fire demand.

Total without Fire Demand = 0.75 cfs

Influent = 2.5 MGD = 3.87 cfs

Left Over to Other Neighborhoods = 3.12 cfs

Distribute 50/50 to two outflow nodes = 1.56 cfs (arbitrary for this problem – would be based on known "downstream" requirements).

Steps for the Hardy-Cross Method

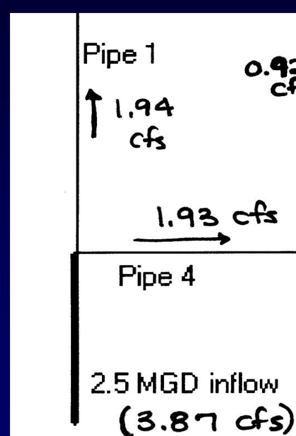
4. Assume internally consistent distribution of flow, i.e., at any given node and for the overall water distribution system:

Σ flow entering node = Σ flow leaving node

Steps for the Hardy-Cross Method

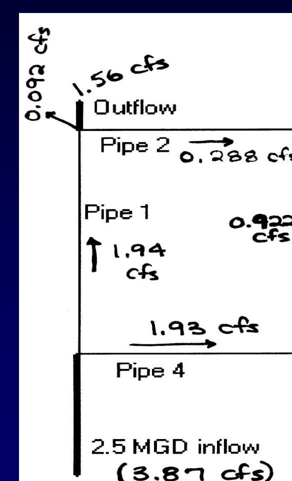
5. For the inflow node, split the flow among the pipes leaving that node (there will be no additional outflow since no water has been used by the neighborhood as yet).

Inflow = Σ Outflows



Steps for the Hardy-Cross Method

6. For each of the pipes leaving the inflow node, put the water demand for that street at the node at the end of the pipe.



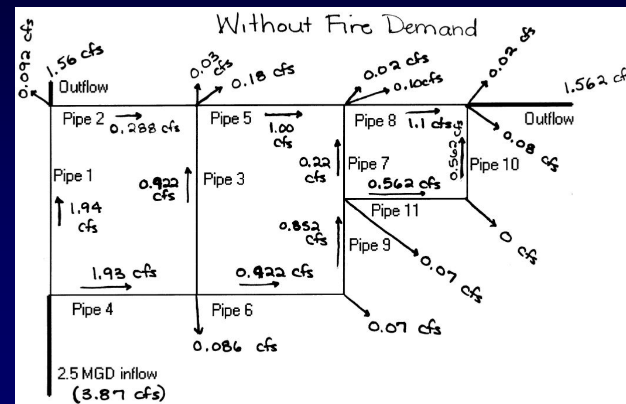
Steps for the Hardy-Cross Method

7. For the above node and the next pipes in the distribution system, subtract the water used on the street (and aggregated at the node) from the water flowing down the pipe. Pass the remaining water along to one or more of the pipes connected to that node.

8. Repeat Steps 6 and 7 for each pipe and node in the distribution system. The calculation can be checked by seeing if the total water outflow from the system equals the total inflow to the system, as well as checking each node to see if inflow equals outflow.

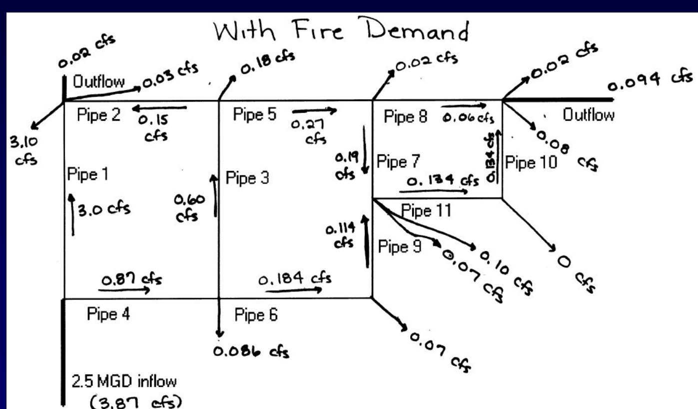
Steps for the Hardy-Cross Method

9a. Check each node to see if inflow equals outflow.



Steps for the Hardy-Cross Method

9b. Check each node to see if inflow equals outflow.



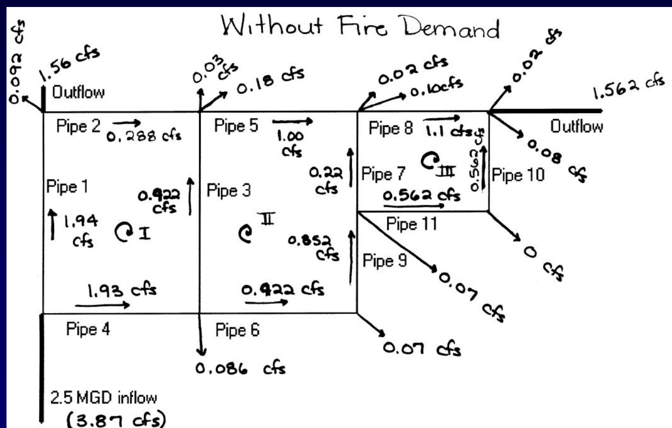
Steps for the Hardy-Cross Method

10. Select initial pipe sizes (assume a velocity of 3 ft/sec for normal flow with no fire demand). With a known/assumed flow and an assumed velocity, use the continuity equation (Q = VA) to calculate the cross-sectional area of flow. (when conducting computer design, set diameters to minimum allowable diameters for each type of neighborhood according to local regulations)

Pipe Number	Flow (ft^3/sec)	Velocity (ft/sec)	Area (ft^2)	Diameter (ft)	Diameter (in)	Actual D (in)
1	1.94	3.0	0.65	0.91	10.9	12
2	0.288	3.0	0.10	0.35	4.2	6
3	0.922	3.0	0.31	0.63	7.5	8
4	1.930	3.0	0.64	0.91	10.9	12
5	1.000	3.0	0.33	0.65	7.8	8
6	0.922	3.0	0.31	0.63	7.5	8
7	0.220	3.0	0.07	0.31	3.7	4
8	1.100	3.0	0.37	0.68	8.2	10
9	0.852	3.0	0.28	0.60	7.2	8
10	0.562	3.0	0.19	0.49	5.9	6
11	0.562	3.0	0.19	0.49	5.9	6

Steps for the Hardy-Cross Method

11. Determine the convention for flow. Generally, clockwise flows are positive and counter-clockwise flows are negative.



Steps for the Hardy-Cross Method

12. Paying attention to sign (+/-), compute the head loss in each element/pipe of the system (such as by using Darcy-Weisbach or Hazen-Williams).

Hazen – Williams

$$h_L = L\left(\frac{Q}{0.432CD^{2.63}}\right)^{1.85}$$

Darcy – Weisbach

$$h_L = f\frac{L}{D}\left(\frac{V^2}{2g}\right)$$

Steps for the Hardy-Cross Method

13. Compute the sum of the head losses around each loop (carrying the appropriate sign throughout the calculation).

14. Compute the quantity, head loss/flow (h_L/Q), for each element/pipe (note that the signs cancel out, leaving a positive number).

15. Compute the sum of the (h_L/Q)s for each loop.

Steps for the Hardy-Cross Method

16. Compute the correction for each loop.

$$\Delta Q = \frac{-\sum_{loop} h_L}{n\sum_{loop}\frac{h_L}{Q}}$$

where n = 1.85

Steps for the Hardy-Cross Method

17. Apply the correction for each pipe in the loop that is not shared with another loop.

$$Q_1 = Q_0 + \Delta Q$$

18. For those pipes that are shared, apply the following correction equation (continuing to carry all the appropriate signs on the flow):

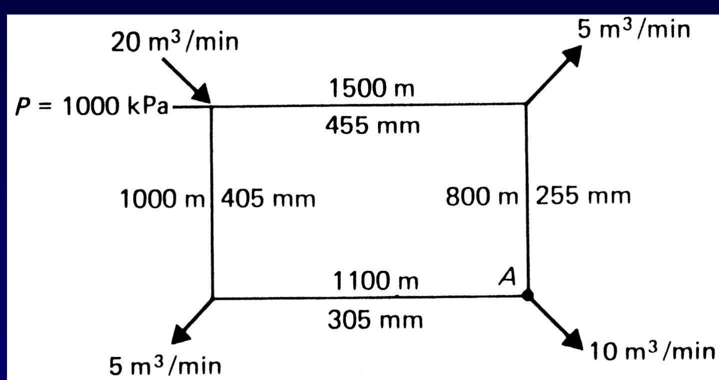
$$Q_1 = Q_0 + \Delta Q_{loop\ in} - \Delta Q_{shared\ loop}$$

Steps for the Hardy-Cross Method

19. Reiterate until corrections are sufficiently small (10 – 15% or less of smallest flow in system), or until oscillation occurs.
20. Calculate velocities in each pipe and compare to standards to ensure that sufficient velocity (and pressure) are available in each pipe. Adjust pipe sizes to reduce or increase velocities as needed.
21. Repeat all the above steps until a satisfactory solution is obtained.
22. Apply fire flow and other conditions that may be critical and reevaluate.

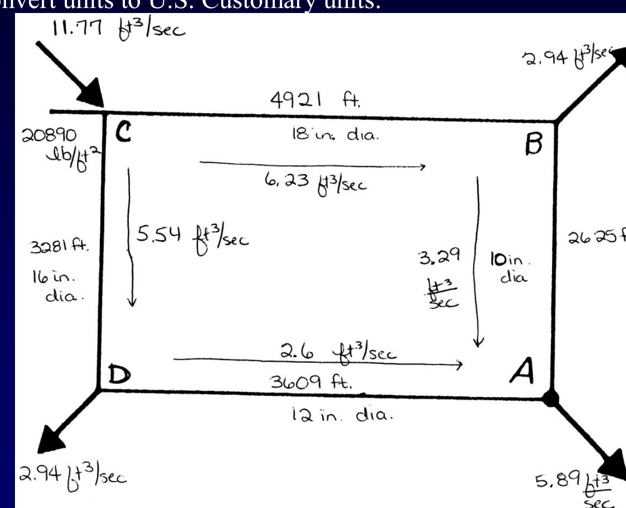
Example for the Hardy-Cross Method

(From McGhee, *Water Supply and Sewerage, Sixth Edition*)



Example for the Hardy-Cross Method

Convert units to U.S. Customary units:



Example for the Hardy-Cross Method

- Insert data into spreadsheet for Hardy-Cross (solve using Hazen-Williams).
- ASSUME: Pipes are 20-year old cast iron, so C = 100.

Pipe Section	Pipe Length (ft)	Pipe Diameter (in)	$Flow_0$ (ft^3/sec)
BC	4921	18	6.23
CD	3281	16	-5.54
DA	3609	12	-2.6
AB	2625	10	3.29

Steps for the Hardy-Cross Method

- Paying attention to sign (+/-), compute the head loss in each element/pipe of the system by using Hazen-Williams (check that the sign for the head loss is the same as the sign for the flow).

$$Hazen-Williams$$

$$h_L = L\left(\frac{Q}{0.432CD^{2.63}}\right)^{1.85}$$

Example for the Hardy-Cross Method

- Calculate head loss using Hazen-Williams.

Pipe Section	Pipe Length (ft)	Pipe Diameter (in)	$Flow_0$ (ft^3/sec)	h_L (ft)
BC	4921	18	6.23	19.03
CD	3281	16	-5.54	-18.11
DA	3609	12	-2.6	-19.93
AB	2625	10	3.29	54.39

Example for the Hardy-Cross Method

- Calculate h_L/Q for each pipe (all of these ratios have positive signs, as the negative values for h_L and Q cancel out).

Pipe Section	$Flow_0$ (ft^3/sec)	h_L (ft)	h_L/Q (sec/ft^2)
BC	6.23	19.03	3.05
CD	-5.54	-18.11	3.27
DA	-2.6	-19.93	7.66
AB	3.29	54.39	16.53

Example for the Hardy-Cross Method

- Calculate head loss using Hazen-Williams and column totals:

Pipe Section	h_L (ft)	h_L/Q (sec/ft^2)
BC	19.03	3.05
CD	-18.11	3.27
DA	-19.93	7.66
AB	54.39	16.53
	$\Sigma h_L = 35.38$	$\Sigma(h_L/Q) = 30.51$

Example for the Hardy-Cross Method

- Calculate the correction factor for each pipe in the loop.

$$\Delta Q = \frac{-\sum_{loop} h_L}{n \sum_{loop} \frac{h_L}{Q}} \quad \text{where } n = 1.85$$

= -(35.38)/1.85(30.51) = -0.627

Example for the Hardy-Cross Method

- Calculate the new flows for each pipe using the following equation:

$Q_1 = Q_0 + \Delta Q$

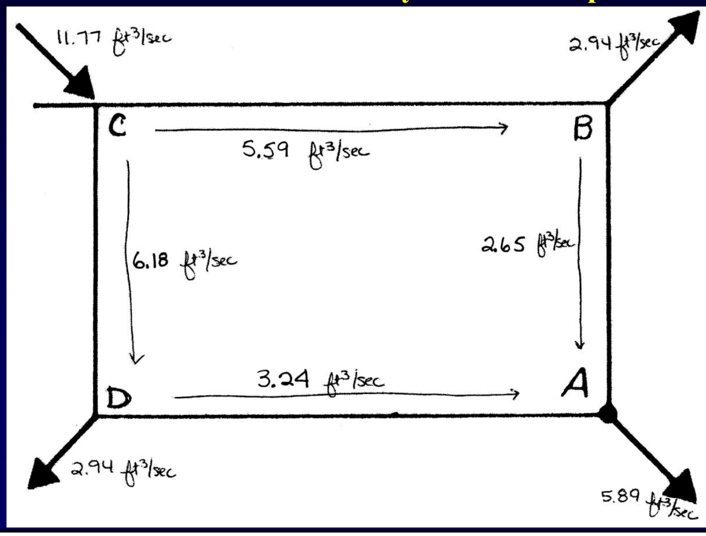
Pipe Section	$Flow_0$ (ft^3/sec)	ΔQ (ft^3/sec)	$Flow_1$ (ft^3/sec)
BC	6.23	-0.627	5.60
CD	-5.54	-0.627	-6.17
DA	-2.6	-0.627	-3.23
AB	3.29	-0.627	2.66

Example for the Hardy-Cross Method

HARDY CROSS METHOD FOR WATER SUPPLY DISTRIBUTION

Pipe Section	Pipe Length (ft)	Pipe Diameter (in)	Flow (ft^3/sec)	H_L (ft)	H_L/Q (sec/ft^2)	$n\Sigma(H_L/Q)$ (sec/ft^2)	$\Sigma(H_L)$ (ft)	ΔQ (ft^3/sec)	Flow (ft^3/sec)
Trial I			$Flow_0$						$Flow_1$
BC	4921	18	6.2	19.03	3.05	56.46	35.38	-0.627	5.60
CD	3281	16	-5.5	-18.11	3.27			-0.627	-6.17
DA	3609	12	-2.6	-19.93	7.66			-0.627	-3.23
BA	2625	10	3.3	54.39	16.53			-0.627	2.66
Trial 2			$Flow_1$						$Flow_2$
BC	4921	18	5.60	15.64	2.79	54.38	0.64	-0.012	5.59
CD	3281	16	-6.17	-22.08	3.58			-0.012	-6.18
DA	3609	12	-3.23	-29.71	9.21			-0.012	-3.24
BA	2625	10	2.66	36.79	13.81			-0.012	2.65
Trial 3			$Flow_2$						$Flow_3$
BC	4921	18	5.59	15.58	2.79	54.34	0.00	0.000	5.59
CD	3281	16	-6.18	-22.16	3.59			0.000	-6.18
DA	3609	12	-3.24	-29.91	9.24			0.000	-3.24
BA	2625	10	2.65	36.49	13.76			0.000	2.65

Final Flows for the Hardy-Cross Example



Pressure Water Distribution System

The pressure at any node can be calculated by starting with a known pressure at one node and subtracting the absolute values of the head losses along the links in the direction of flow.

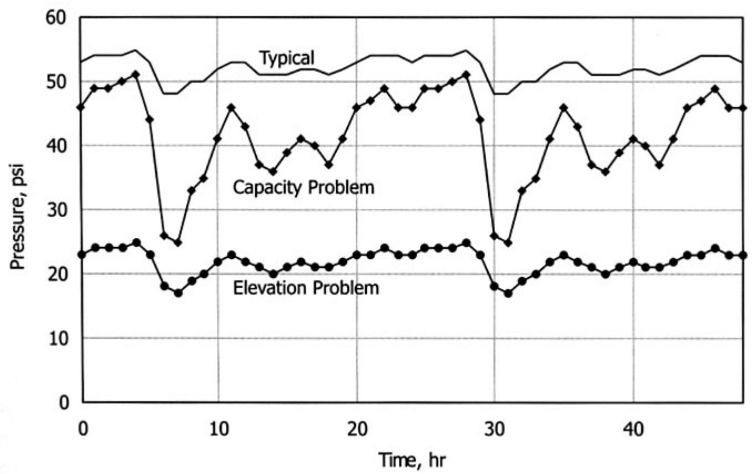
In this example, assume that the pressure head at node C is 100 ft. and the pressure head at node A is desired.

There are two paths between the known and unknown nodes for this example and both should be examined: CB and BA or CD and DA.

In the first case: 100 ft. – 15.58 ft. – 36.49 ft. = 47.93 ft.

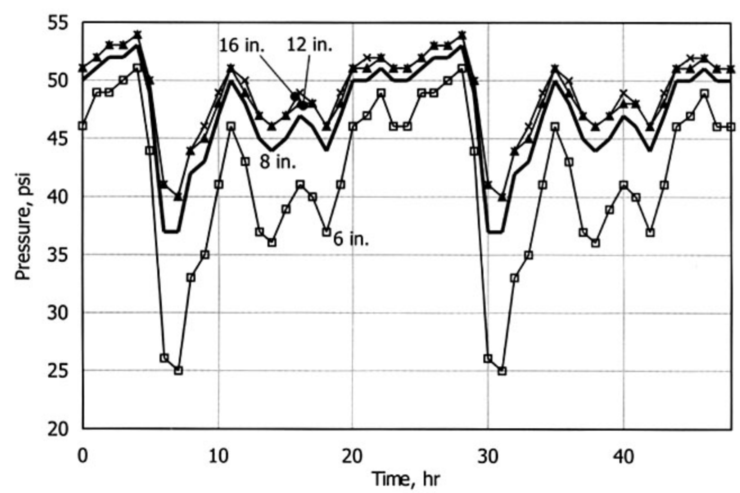
And in the second: 100 ft. – 22.16 ft. – 29.91 ft. = 47.93 ft.

Extended period simulation (EPS) runs showing low pressure due to elevation or system capacity problem



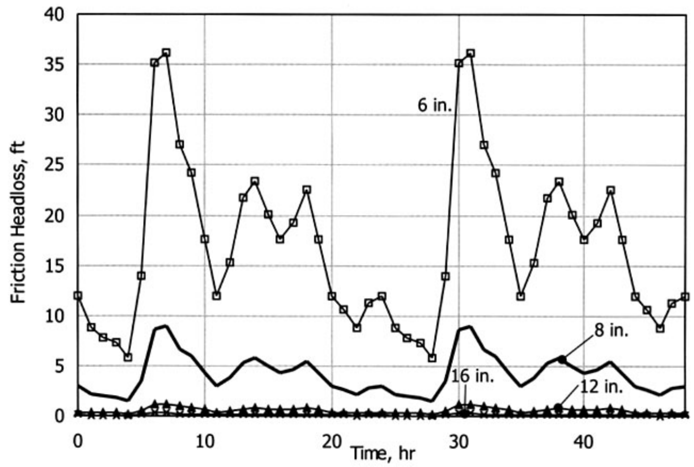
(Walski, *et al.* 2001 figure 7.3)

Pressure comparison for 6-, 8-, 12-, and 16- inch pipes



(Walski, *et al.* 2001 figure 7.4)

Head loss comparison for 6-, 8-, 12-, and 16- inch pipes

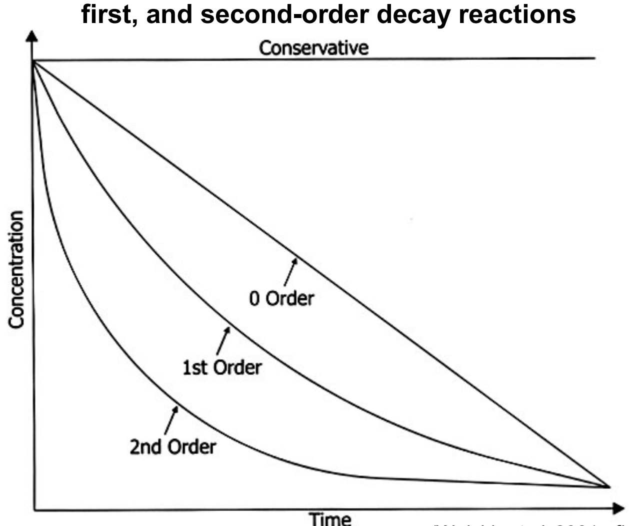


(Walski, *et al.* 2001 figure 7.5)

Chemical Reactions in Water Distribution Systems

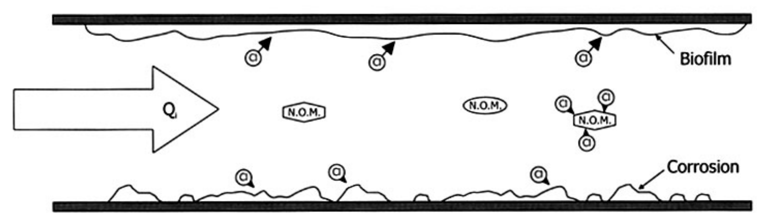
- Many of the water distribution models include water quality subcomponents.
- These are used to determine the age of the water in the distribution system and the mixing of water from different sources at the different locations.
- This information is used to calculate the resulting concentrations of conservative and nonconservative compounds in the water.
- Knowledge of the fate of disinfectants in the distribution systems is very important to ensure safe drinking water: we need to ensure that sufficient concentrations of the disinfectants exist at all locations in the system.

Conceptual illustration of concentration vs. time for zero, first, and second-order decay reactions



(Walski, *et al.* 2001 figure 2.23)

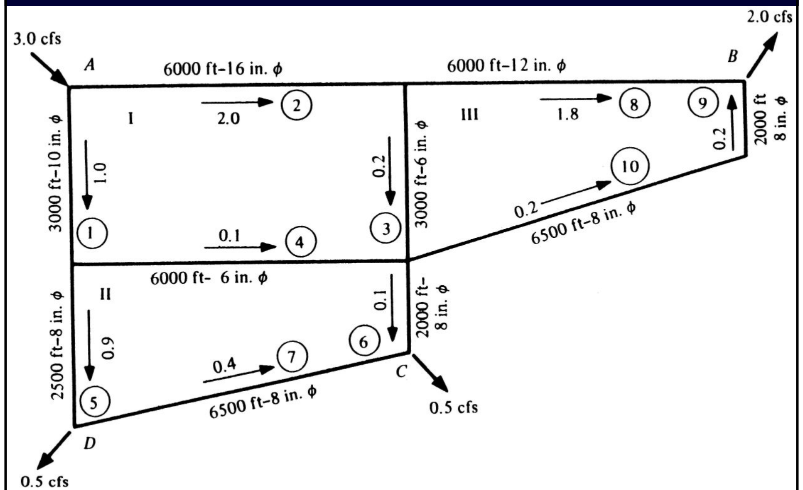
Disinfectant reactions occurring within a typical distribution system pipe



N.O.M. = natural organic matter

(Walski, *et al.* 2001 figure 2.24)

Another Example for the Hardy-Cross Method



EPANet Water Distribution Model